EFFECTS OF CLOUDS ON THE RADIATIVE HEAT EXCHANGE IN THE ATMOSPHERE

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RESUMEN:

Las nubes son los principales reguladores del flujo radiativo. A su vez las nubes mismas están sujetas a efectos directos e indirectos de la radiación. Los efectos directos se producen mediante absorción y emisión de radiación por las nubes. Los efectos indirectos se producen mediante el calentamiento de la superficie terrestre por la radiación solar, lo cual define el transporte de humedad hacia la atmósfera y la formación de las nubes.

En este trabajo se presentan algunos resultados de investigaciones realizados sobre los flujos radiativos bajo condiciones de nubosidad. Se muestran algunos métodos simples aproximados de cálculos y las regularidades del comportamiento del campo de nubes dependiente de los parámetros básicos, así como los efectos de retroalimentación.

Este trabajo se pretende utilizar en los problemas de dinámica de la atmósfera de gran escala y en el programa del GARP.

ABSTRACT

Clouds are the main regulators of radiative fluxes. They themselves are subjected to both direct and indirect effects of radiation. The former are caused by absorption and emission of radiation by the clouds. As for the latter heating of the Earth’s surface by the Sun ultimately defines the moisture flow into the atmosphere and cloud formation.

This paper presents some results of investigations of radiative fluxes under cloud conditions – simple, approximate methods of computation, regularities of the behaviour depending on basic parameters, feedback effects. The work is intended for use in the problems of large-scale dynamics of the atmosphere, and in the GARP program.

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1. STRATIFIED CLOUDS

The first means to simplify computations of radiative fluxes is the direct computation of the integral fluxes instead of the spectral ones with the subsequent integration by wave lengths. For the longwave (thermal) radiation this procedure is well known and has been used for a long time (Feigelson, 1970).

Direct computation of integral fluxes of near infrared solar radiation is also a commonly used method for cloudless conditions. This method can also be applied for cloudy atmosphere. (See Section, 1.2, Feigelson et al., 1972, 1973).

For computation of integral fluxes the following integral transmission function should be prepared:

\[
P^{(i)}(m_1, \ldots, m_n) = \frac{\int_0^\infty f^{(i)}_\lambda \prod_{j=1}^n P_{j,}^{(i)}(m_j) \, d\lambda}{\int_0^\infty f^{(i)}_\lambda \, d\lambda} \quad \ldots \quad i = 1,2 \tag{1}\]

Here \(m_j\) is the content of the \(j\)-s absorbing matter on the radiation path. \(P_{j,}^{(i)}(m_j)\) – a respective spectral transmission function, \(f^{(1)}_\lambda\) – the spectral solar constant; \(f^{(2)}_\lambda = \beta_\lambda(\overline{T})\) – the Planck function; \(\overline{T}\) - the mean temperature of the troposphere.

For the troposphere it is sufficient to take into consideration three absorbing substances: water vapor \((j = 1)\), carbon dioxide \((j = 2)\) and water droplets (crystals) of clouds \((j = 3)\).

The function \(P^{(2)}(m_1, m_2)\) is commonly known. Precise tables of the function

\[
D^{(2)}(m_1, m_2) = 2^{\frac{1}{2}} \int_0^1 P^{(2)} \left( \frac{m_1}{\mu}; \frac{m_2}{\mu} \right) \mu \; d\mu \tag{2}
\]

were published by Niiilisk et al., 1969.

The function \(P^{(1)}(m_1, m_2)\) is given by Yevseeva et al., 1970. We have proposed functions \(P^{(1)}(m_1; O; m_3)\) (Feigelson et al., 1973) and \(P^{(2)}(m_1, m_2, m_3)\) (Feigelson, 1970). The functions \(P^{(1)}(m_1, m_2)\)
and \( P^{(1)}(m_1; O; O) \) differ not more than by 10% within the tropospheric conditions.

Functions \( P^{(i)}(m_1, \ldots, m_n) \) allow to consider the cloudy atmosphere as a medium with additional (in respect to atmospheric gases) absorbing and scattering substances — water in droplets and in crystalline states. Such approach allows to give up the a priori assumption on the "black" radiation of cloud boundaries what in turns, permits to construct continuous vertical profiles of radiative fluxes including the inner parts of the clouds. It makes possible to study radiative effects of weak — obviously not "black" cloudiness—, to describe continuous transitions from cloudless to cloudy conditions.

For the fluxes of IR solar radiation this approach makes possible to take into account scattering and absorption of cloud particles together with the absorption in the bands of atmospheric gases in a single non-complicated algorithm (see Section I.2).

It should be mentioned that utilization of the functions \( P^{(i)}(m_1, \ldots, m_3) \) implies the knowledge of liquid water amount of clouds \( W(Z) \) or the water content:

\[
m_3 = \int_{Z_1}^{Z_2} W(Z) \, dZ
\]

where \( Z_1, Z_2 \) are the levels of cloud boundaries. It is difficult to compute the function \( W(Z) \), since there is a minor difference between the computed humidity and saturation. At the same time, the parameter \( m_3 \) is used not only for the functions \( P^{(i)} \) but mainly defines the cloud albedo (see I.2). Thus parameterization of the quantity \( W \) or \( m_3 \) is required. Their connection with temperature is doubtless but with thickness \( (Z_2 - Z_1) \) is observed (Feigelson, 1970).

Of a certain difficulty in the use of functions \( P^{(i)} \) is the spectral dependence of variable \( m_1, m_2 \) defined by the formula

\[
m_{\lambda,j} (Z) = \int_{Z}^{Z_2} \rho_j (Z) \left( \frac{P(Z)}{P_o} \right)^{n_{\lambda}^{(i)}} dZ
\]
Here $\rho_j(Z)$ are the densities of water vapor and CO$_2$. In case of $i = 1$, the exponent $n^{(1)} = 1$ is assumed on the average; with $i = 2$ the mean values are $n^{(1)} = 0.5$ and $n^{(2)} = 0.8$.

I.1. Longwave radiation

To describe longwave flux divergence $- R(Z)$ in the models of a stratified atmosphere it is common to use an equation like the following one:

$$
R(Z) = \frac{d}{dZ} \left\{ B_S D(m) - B_H D(m^x - m) + \int_0^H B(Z') \frac{dD(m - m'/)}{dZ'} \right\} dZ' \ldots
$$

(5)

Here $B(Z) = \sigma T^d(Z)$; $B_S$ and $B_H$ are the emissions of "black" or "gray" boundaries of the layer under consideration (O, H). The quantities $m^x$ and $m'$ are determined by the formula (4) with $Z = H$ and $Z = Z'$ respectively. For the sake of simplicity here and further we consider the case of one absorbing substance.

To simplify equation 5 a sequence of approximate methods can be proposed, according to the absorptivity of the medium.

a) The diffusion approximation, corresponding to the assumption: the function $D(m)$ is $\delta$ - like, i.e., an extremely high absorptivity of the atmosphere;

b) approximation "1", corresponding to the assumption on $\delta$ - like module of the first derivative of the function $D(m)$, i.e. somewhat less but also sufficient optical density of the atmosphere;

c) approximation "2", corresponding to the assumption on the $\delta$ - like second derivative of $D(m)$, i.e. still lesser absorptivity of the medium;

d) approximation applied under the conditions of great transparency of the atmosphere when $D(m) \approx 1$-am.

The function $f(x)$ is called $\delta$ - like here, if $\int_0^H \varphi(x) f(|x - x_0|) dx = $
\[ \varphi(x_0) \int_0^H f(|x - x_0|) \, dx. \]

Approximation (a) is commonly known. In case (b) the fluxes are expressed by

\[
F^{\uparrow}(Z) = B_H D(m) + B(Z) \, [1-D(m)]
\]
\[
F^{\downarrow}(Z) = B_H D(m^x - m) + B(Z) \, [1-D(m^x - m)]
\]

(6)

\[
R(Z) = - \frac{d}{dZ} [F^{\uparrow}(Z) - F^{\downarrow}(Z)]
\]

(7)

For cases (c) and (d) simple expressions can be given for the flux divergence:

(c): \[ R(Z) = [B_s - B(Z)] \frac{dD(m)}{dZ} + [B(Z) - B_H] \frac{dD(m^x - m)}{dZ} \]

(8)

(d): \[ R(Z) = - \alpha \rho(Z) \, [B_s - 2B(Z) + B_H] \]

(9)

The last expression is a version of the Newton’s law: the flux divergence is proportional to the difference of the temperatures of the characteristic layers.

Comparison of approximations (b) – (d) with computations made by the formula (5) and with more precise spectral computations is described in detail by Feigelson (1970) and in the “Heat Exchange in the Atmosphere” (1972). The error of the approximation (b) in the fluxes for the standard atmosphere is on the average less than 10%, reaching 20-30% only when computing \( F^{\uparrow}(Z) \) with \( Z \approx H \). This error can be decreased down to 1-2% if we replace the function \( D(m) \) by its mean value \( \bar{D}(m) \) for the \((O, Z)\) layer:

\[
\bar{D}(m) = \frac{1}{m} \int_0^m D(m - m') \, dm'
\]

(10)

The accuracy of approximations (b) and (c) is about 20% on the average for flux divergences.
Let us dwell upon the radiative properties of cloudy layers, described in detail in the two above mentioned monographs.

A stratified cloud is an emitter comparable with the atmosphere on the whole as an emitter. At the same time this emitter possesses peculiarities. It is strictly localized in space and can be identified with a thermal sink near the upper boundary within the cloud (Fig. 1). The heat loss of this sink is almost the same, as of the whole atmosphere outside the cloud.

The lowest part of the cloud is heated by the thermal radiation. This heat source is small as compared with the sink, but it is comparable with the cooling out of a layer of the same thickness and level in cloudless conditions.

In the cloudy atmosphere the weight of different layers in the total radiative cooling out is not proportional to the thickness of the layers. One may neglect the cooling out of the whole under cloudy layer and of the most part of the cloudy one. But the heat loss by the narrow upper part of the cloud may not be ignored. The latter, together with the heat loss by above cloudy atmosphere and heat source near the lower boundary of the cloud, comprises nearly all radiation heat loss of the atmosphere.

With multilayered cloudiness each cloudy layer is a thermal sink much more essential than the intercloudy space; the heat loss by the upper cloudy layer is specially large.

### Table 1

<table>
<thead>
<tr>
<th>Nature of layers</th>
<th>Number of cloudy layers</th>
<th>One (1-2 km)</th>
<th>Two (1-2 km)</th>
<th>Three (5-6 km)</th>
<th>Three (7-8 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undercloudy and intercloudy</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cloudy</td>
<td>43</td>
<td>62</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overcloudy</td>
<td>57</td>
<td>35</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table I gives the heat loss in percent for different layers with respect to the total heat loss by the atmosphere.

The boundaries of the cloudy layers are given in brackets. The expressions of the cloudy flux divergence are given below under the conditions of approximation (b) (Feigelson, 1970):

Maximum heat loss \( R(Z_2 - \Delta Z_2) \) under the upper boundary of the cloud and maximum heating above the lower one \( R(Z_1 + \Delta Z_1) \) are described by the expressions:

\[
R(Z_2 - \Delta Z_2) \approx -\sqrt{\alpha} \left| \frac{dW}{dZ_2} \right| B(Z_2) \ D^{(2)}(m_1^x - m_{1,2}^x; m_2^x - m_{2,2}^x; 0)
\]

(11)

\[
R(Z_1 + \Delta Z_1) \approx \sqrt{\alpha} \left| \frac{dW}{dZ_1} \right| B(Z_1) \ D^{(2)}(m_{1,1}; m_{2,1})
\]

(12)

Here \( m_{j,1}; m_{j,2}; m_j^x \) (\( j = 1,2 \)) are determined by the formula (4) with \( Z = Z_1 ; Z_2 \) and \( H \) respectively \( \frac{dW}{dZ_i} = \frac{dW}{dZ} \) by \( Z = Z_i \) (\( i = 1,2 \)); \( \alpha = 1000 - 2000 \ \text{cm}^2/\text{g} \) is the absorption coefficient of water droplets averaged over their dimensions and over the spectrum of wave-length.

Radiation peaks are formed at the distances \( \Delta Z_i \) from cloud boundaries, where

\[
\Delta Z_i \approx \left( \frac{1}{\sqrt{\alpha} \left| \frac{dW}{dZ_i} \right|} \right)^{-1} \quad i = 1,2
\]

(13)

From (11) – (13) follow expressions for the boundary layers flux divergences:

\[
R_1 = R(\Delta Z_1) = [B_S - B(Z_1)] \ D^{(2)}(m_{1,1}; m_{2,1}; 0)
\]

(14)

\[
R_2 = R(\Delta Z_2) = -B(Z_2)D^{(2)}(m_1^x - m_{1,2}^x; m_2^x - m_{2,2}^x; 0)
\]

(15)
and for the cloud on the whole:

\[ R_{c1} = R_1 + R_2 \]  \hspace{1cm} (16) 

since for the inner parts of the cloudy layer the flux divergence is small.

Equations (11) – (16) being compared with the results of computations by the formula (5) give a reasonable error: of the order of 10% – 20% – formulas (14) – (16) and up to 100% for (11) – (13). The flux divergence for the whole overcloudy layer within the same approximation can be represented by the formula

\[ R(H - Z_2) = - B(H) \left[ 1 - \bar{D}^{(2)}(m_1 - m_{1,2}; m_2^x - m_{2,2}; O) \right] \]  \hspace{1cm} (17) 

where the function \( \bar{D}^{(2)} \) is defined by the formula (10). Finally, for the undercloudy layer we can assume the flux divergence equal to zero at least in the case of \( Z_1 <5-6 \text{ km} \).

Formulas (11) – (16) show the behaviour of the radiation properties of the cloudy layer depending on the parameters of this layer and an extra-cloudy atmosphere.

In conclusion, we can mention that the assumption on black radiation of cloud boundaries is fairly precise with \( W \geq 0.03 \frac{g}{m^3} \) for computation of the flux divergence for the whole cloud or whole overcloudy atmosphere and for the levels \( Z << Z_1 \) and \( Z >> Z_2 \).

Possibilities of utilization of the function \( D^{(2)}(m_1, m_2, m_3) \) are shown on Fig. 1 on the left side. Here the continuous curve presents mean data on radiative cooling out into the St, Sc clouds obtained from 25 sets of aircraft measurements (Feigelson, 1970, "Heat Exchange in the Atmosphere", 1972). Fluxes within the cloud were measured with the interval of \( \Delta Z = 50 \text{ m} \) and close to the upper boundary with \( \Delta Z = 100 \text{ m} \). The dotted curve on the left side is the computation by the formula (5) with the utilization of \( D^{(2)}(m_1, m_2, m_3) \) and the mean for the aircraft data: \( Z_2 - Z_1 = 400 \text{ m}; \ Z_2 = \)
700 m; $W = 0.2 \text{ g/m}^3$, with mean profiles of temperature and humidity.

I.2. Short-wave radiation

The problem of computing solar radiation fluxes in clouds requires an account of multiple scattering and remains within the classical transfer theory only until the bands of atmospheric gas absorption are not considered. The transmission functions for small spectral intervals $\Delta \lambda$ appear to be non-exponential, owing to the discrete structure of gas spectra. The concept of the absorption coefficient loses sense and the transfer equation ceases to be applicable.

To solve the problem on the transfer of IR solar radiation in clouds with due account of multiple scattering, absorption on drops and absorption by atmospheric gases we use the ideology of photon paths. (Van de Hulst et al., 1963).

Let the probability density of photon’s path lengths $-Y_\text{(A)} (\ell)$ \(Y_\text{(T)}(\ell)\) ((A)– for reflection and (T)– for transmission) be given for a homogeneous cloud layer under the condition of pure scattering. Then the normalized fluxes of reflected (A) and transmitted (T) radiation (albedo and transmissivity) with a given transmission function are of the following form:

\[
A_{\Delta \lambda} = \int_{0}^{\infty} Y_\text{(A)} (\ell) P_{\Delta \lambda} (\rho \ell) \, d\ell 
\]  \hspace{1cm} (18)

\[
T_{\Delta \lambda} = \int_{0}^{\infty} Y_\text{(T)} (\ell) P_{\Delta \lambda} (\rho \ell) \, d\ell 
\]  \hspace{1cm} (19)

The absorptivity is equal to

\[
\pi_{\Delta \lambda} = 1 - A_{\Delta \lambda} - T_{\Delta \lambda} 
\]  \hspace{1cm} (20)

With $P_{\Delta \lambda} (\rho \ell) = 1$ formulas (18), (19) represent albedo and transmission under the conditions of pure scattering.
\[ A_{\Delta \lambda} = \int_0^\infty Y_A (\xi) \, d\xi \]

\[ (T_{\Delta \lambda}) \]  

The dependence of the distributions \( Y_A (\xi) \) on optical parameters \( (T) \) of the cloud, its thickness and zenith angle of the Sun \( \xi \) is analyzed by Feigelson et al. (1972) using the Monte-Carlo method. They give the \( A \) and \( \pi \) values for all the bands of water vapor with \( \lambda \leq 3.6 \) mkm, and integral values of \( A \) and \( \pi \), as well. Absorption is negligible small and its respective albedo is close to that in the case of pure scattering with \( \lambda < 0.9 \) mkm and with mean humidity of the cloud \( \rho_1 < 5 \, g/m^3 \). Water vapor absorption appears to be noticeable in the whole considered interval of wave-length if the optical thickness of the cloud \( \tau \geq 20 \) and \( \rho_1 \geq 5 \, g/m^3 \).

The role of water droplets in absorption is displayed with \( s \geq 1.3 \) mkm. On the average with \( s \leq 1.5 \) mkm the main absorber is the water vapor and with \( s \geq 2.3 \) mkm the effect of water droplets prevails, absorbing (with \( W \geq 0.1 \, g/m^3 \)) the whole solar radiation, (with \( s > 2.3 \) mkm) incoming to the cloud. For the band \( \Omega \) (1.66 \leq s \leq 2.08 mkm) water vapor and water droplets play nearly the same role in formation of the reflection and absorption.

Integral albedo of the cloudy layer poorly depends on absorption. The dependence of albedo on the Sun position and on the form of the scattering function is more essential. At last, the dependence on optical thickness \( \tau \) of the cloud is the main one. Here

\[ \tau = \sigma_o \, (Z_2 - Z_1) \, W = \sigma_o \, m_3 \]  

(22)

and \( \sigma_o \) is the scattering coefficient of the order of 1000-3000 cm²/g. Table 2 gives integral albedo and the absorptivity of clouds. Computations were made with the scattering function \( \gamma_S(\varphi) \) of a polydispersal cloud by \( s = 0.7 \) mkm (Deirmenjian, 1969), for the
spectral interval $0.4 \leq s \leq 1.7$ mkm. The function $\gamma_s(\varphi)$ by $s = 27$ mkm was used for $s > 1.7$ mkm. The albedo of the pure scattering $-A_s$ is given with both functions $\gamma_s(\varphi)$, the numbers in brackets by $s = 2.7$.

These estimates allow to believe that in the problems of large-scale dynamics albedo values of pure scattering may be used. Table 2 permits also to take into account variations of $A$ with the optical thickness of the clouds and with the Sun position. Dependence on these factors is a decisive factor while analyzing feedback effects (see Section III).

Aircraft measurements of albedo of clouds Se, St, accomplished by Goisa et al. ("Heat Exchange in the Atmosphere", 1972), supplied the data, given on Fig. 2 and the empirical formula:

$$A = 1 - e^{-0.4 s^4 \sqrt{m3}}$$

on the average at $40^\circ \leq \xi \leq 85^\circ$. Fig. 2 presents also the $A$ values computed by Feigelson et al. (1972) with $\xi = 60^\circ$ and $70^\circ$ and $s = 50$ km$^{-1}$.

From the measurements and the computations the absorptivity of the cloudy layers poorly depends on the Sun position.

Coming back to the Fig. 1 (right side) we can compare measurements and computations of heating caused by the Sun inside the cloudy layer. Discrepancies near the upper boundary, are caused evidently, by averaging in flights over cloudy and extra cloudy space.

We come across another difficulty with estimations of solar radiation absorption in heterogeneous cloudy atmosphere. Non-exponential transmission functions do not agree with the principle of superposition,

$$P_{\Delta\lambda} (m_1 + m_2) \neq P_{\Delta\lambda} (m_1) P_{\Delta\lambda} (m_2)$$

and a consecutive radiation transfer in the system of layers can not be considered. The idea for overcoming this difficulty can be first
analyzed on the example of cloudless atmosphere with one absorbing substance, the scattering of IR solar radiation being ignored.

In this case the relative fluxes are of the following form:

\[
F_\downarrow (Z) = P \left[ \frac{1}{\mu_0} \left( m^x - m (Z) \right) \right] \\
F_\uparrow (Z) = A_S D \left[ \frac{1}{\mu_0} m^x + \frac{1}{\mu} m (Z) \right]
\] (24) (25)

where \( A_S \) is the albedo of the isotropic underlying surface; \( \mu_0 = \cos \xi \). \( \mu = \cos \theta \); \( \bar{\theta} \) – mean angle of reflection.

The function \( m(Z) \) is an additive one (see eq. (4)), which allows to distinguish layers with peculiarities of the content of the absorbing substance, then to compute the summary value of \( m(Z) \) and finally, the fluxes by formulae (24), (25).

Scattering in the clouds leads to a lengthening of photon paths. Knowing the latter we can determine the fluxes by means of the just mentioned method.

Feigelson et al. (1973) considered the case of integral fluxes of IR solar radiation by these patterns under the following assumptions:

1. Beyond the clouds only attenuation of solar radiation caused by absorption by atmospheric gases and by absorption and scattering by aerosol are taken into account.

   Within the clouds multiple scattering and absorption by particles as well as absorption by atmospheric gases are taken into consideration.

2. The atmosphere is assumed to be vertically stratified with the variable in height densities \( \rho_j (Z) \) of the absorbing gases and the pressure \( p(Z) \). Cloudy layers are assumed to be homogeneous with mean densities \( \rho_j, c \ell \) and the pressure \( p_c \ell \).

   For the photon which passed the path \( \ell \) in the cloud, one gets:

\[
m_{j,c \ell} = \rho_{j,c \ell} \left( \frac{p_{c \ell}}{p_0} \right)^n \ell \text{ with } n = 0 \text{ for } j = 3.
\] (26)

3. The spectral dependence of the scattering within the cloud is ignored with \( \lambda < 3 \text{ mkm} \).
The expressions of fluxes in the presence of a cloud has the following form (the absorption of CO₂ being ignored and with \( A_S = 0 \)).

\[
F_{\uparrow \downarrow}(Z) = \int_0^1 P^{(1)} \left\{ [M_1 + \ell \rho_1, c \ell \left( \frac{P_c \ell}{P_o} \right)^n ]; \ \ell w \right\} Y_A (\ell) d\ell \quad (27)
\]

where \( M \) – is the total content of H₂O gas outside the cloudy layer.

The formula (27) is applicable with \( Z \geq Z_2 \) and \( Z \leq Z_1 \). The computation of \( M \) requires the knowledge of the mean direction of the reflected and transmitted light, or (non-isotropy being ignored) the use of the transmission function for diffuse radiation. The further simplification is achieved by a definition of the effective path \( \ell_{\text{eff}} \ (T) \) of photons from the equation:

\[
\int_0^\infty P^{(1)} \left\{ [M_1 + \ell \rho_1, c \ell \left( \frac{P_c \ell}{P_o} \right)^n ]; \ \ell w \right\} Y_A (\ell) d\ell =
\]

\[
A (T) P^{(1)} \left\{ [M_1 + \ell_{\text{eff}}, (T) \rho_1, c \ell \left( \frac{P_c \ell}{P_o} \right)^n ]; \ \ell_{\text{eff}}, (T) \ W \right\} \quad (28)
\]

If the function \( P^{(1)} (m_1, m_3) \) is known we can define \( \ell_{\text{eff}} (T) \) depending on a large number of variables. Therefore the use of these parameters is justified only in case of weak dependence on the most part of variables. Feigelson et al. (1972), showed that the \( \ell_{\text{eff}}, (T) \) path poorly depends on \( M, \xi, \rho_1, c \ell \left( \frac{P_c \ell}{P_o} \right)^n \) being close to the mean path \( \ell m, (T) \) under the conditions of pure scattering.

\[
\ell m, (T) = \frac{\int_0^\infty Y_A (\ell) \ell d\ell}{\int_0^\infty Y_A (\ell) d\ell} \quad (29)
\]
The dependence of $\xi_{\text{eff}}, A$ on $\xi$ can reach 20%, whereas that on $\rho_1 \, cl_{\text{(T)}} \left( \frac{pe_1 \ell}{p_o} \right)^n$ is even smaller. The dependencies of $\xi_{\text{eff}}, A$ on the optical thickness of the cloudy layer or on its water content appear to be the main.

Fig. 3 gives effective and mean optical paths with disregarding extinction of radiation outside the cloud.

The figure shows that in the transmitted light the effective optical path exceeds several times the optical thickness of the layer. The difference between effective and mean path is rather small. In the reflection regime dependence on $\tau$ is much less and is comparable with the difference between mean and effective ways. Of the same order are variations of $\xi_{\text{eff}}, A$ depending of $\xi$.

For the geometrical path we can write down:

$$\tilde{\xi}_{\text{A, eff}} = \frac{1}{\sigma_w} \varphi(\tau)$$

The function $\varphi(\tau)$ is represented in Fig. 3.

Fluxes and flux divergences of solar radiation in the cloudy atmosphere may be computed in a rather simple way with a reasonable accuracy by the outlined method.

The present estimations of the dependence of $\xi_{\text{eff}}, A$ on the number of layers, their optical properties, the number and content of absorbing substances are being made more precise. A precision is also required for the angular structure of light, reflected or transmitted by cloudy layers.

II. CUMULI CLOUDS

The study of cumuli radiation properties requires an statistical approach.

The statistical structure of radiative fluxes in the process of cumuli development can be studied and parameterized using the continuous
time registration of fluxes on the ground for several hours or spatial registration over hundreds of kilometers during aircraft flights above (or under) the clouds.

A great deal of data of such investigations are represented in the monographs: "Heat Exchange in the Atmosphere" (1972), "Stochastic structure of cloud fields and radiation" (1972) and in the articles by Timanovskaya (1973), Feigelson et al. (1972).

Information on cloudiness was obtained by photographing the reflections of the cloudy sky in semispheric mirrors and also by the analysis of the probability densities of values of direct solar radiation and of readings of optical sensors directed to zenith or to the Sun. The following characteristics of cloudiness were determined.

The total relative cloud amount n, zonal amounts in certain angular zones of the sky among which especially important parameters are: the cloud amount in direction of the Sun n₀ and in the zenith n₀, duration of the sunshine s, probability densities of clouds and gaps by sizes and by number for given size. A correlative and spectral analysis of cloud fields and radiation were made.

The connection between the mean, for 1-2 hours normalized fluxes of the direct $\bar{Y}$ and total $\bar{Q}$ radiation near the Earth’s surface and parameters n or s is linear and can be described by the equations of linear regression of the type:

\[ \bar{Y} = 1.00 - 0.06n \quad (r = 0.81) \]  \hspace{1cm} (30)

\[ \bar{Q} = 1.04 - 0.10n \quad (r = 0.89) \]  \hspace{1cm} (31)

The coefficient of linear correlation r in similar formulae of dependence on s equals 0.95 and 0.98 respectively.

The cloud-amount n here was determined by 3-4 photographs of the cloudy sky for the period of observations of 1-2 hours. The sunshine duration s was defined by the probability density functions of the relative direct radiation flux as a fraction of the observational period, satisfying the conditions $\bar{Y} \geq \epsilon$ where $\bar{Y}$ is the instantaneous flux of solar radiation; $\epsilon = 0.7-0.4$ cal/cm²/min with $\xi = 35-50$. The
fluxes \( \bar{Y} \) and \( \bar{Q} \) in (30), (31) are related to the mean values of fluxes in cloudless conditions with the account of the diurnal change.

Approximation formulae for the dispersion dependencies \( \sigma_Q^2 \) and \( \sigma_Y^2 \) on the parameter \( n \) have the form:

\[
\sigma_Q^2 = 0.1 \sin \left[ \pi \left( n - 0.1 \right) \right] \\
\sigma_Y^2 = 0.15 \sin \left[ \pi \left( n - 0.15 \right) \right]
\]  
(32)  
(33)

The theory of radiation transfer with partially cloudy sky has been developed in the Soviet Union in two ways:

Vainiko (1972), has proposed a three-dimensional equation for the determination of the mean radiation intensity under the assumption of local confusion and global regularity of clouds distribution. Some numerical results were obtained by Avaste et al. (1972), with the simplest assumptions on the involved statistical parameters.

Busygin et al. (1973), applied the Monte-Carlo method to calculate the reflective properties (brightness indicatrix, radiance, albedo) of an individual cumuli depending on its size, optical properties and form.

Having computed the albedo \( A(D) \) of the clouds with different diameters \( D \), these authors determined the relative flux of total radiation near the surface by the formula:

\[
Q = p(\xi) + \left[ 1 - p(\xi) \right] \int_{D_{\text{min}}}^{D_{\text{max}}} \left[ 1 - A(D) \right] f(D) \, dD
\]  
(34)

Here \( p(\xi) \) –is the probability of the line of free vision. \( f(D) \) the distribution of clouds by sizes. These values were defined in the described above experimental works, in the theory represented in the same monographs and also in the works of Kauth et al. (1966) and Plank (1969). Formula (34) disregards multiple light reflection between the clouds. The measured and computed mean fluxes are compared on Figure 4 in dependence on \( n_0 \). The figure gives grounds to believe that the effect on the transmitted and reflected fluxes of the multiple light reflection between the clouds is not too great, as
well as the effect of the diverse forms of the clouds and also that of the solar radiation absorption.

Thus the fluxes of solar radiation under the cumuli are defined, mainly, by the amount of clouds, their distribution by sizes, the scattering coefficient of the cloudy medium, and by the solar zenithal angle.

III. FEED-BACK MECHANISM IN THE PROCESSES OF HEAT AND MOISTURE EXCHANGE

The present part describes and attempt to use the above mentioned approximate methods for the solution of the problem of feed-back mechanism in the process of heat and moisture exchange (Feigelson et al., 1973; Petoukhov, 1974).

In the search for physical processes determining long period variations of weather the attention was often turned to the ocean as the reservoir of heat and to clouds as the regulator of solar radiation reaching the earth surface. (Monin, 1969)

Not only the cloud amount is changed in the total dynamic process, but also their density and with it the reflective, absorbing and emitting capacities. Here we are discussing the possibility of the regulating effects of the clouds on the process of heat and water exchange.

In order to define this effect in its "pure" form and to facilitate the problem it is suggested that the Earth is completely covered by the ocean. The plane model of horizontally homogeneous ocean and atmosphere above it is considered with overcast sky or with statistically homogeneous partial cloudiness.

Upon these conditions the equations of heat and water (humidity) transfer are almost entirely set apart from the equations of motion and can be considered at a given parametric wind velocity. It should be noted that the conclusion about the decisive role of the first equations in the description of the global processes can be found, for instance, in the work by Adem (1962) and in his later works.
The problem is solved both in the linear approximation by the method of small derivations in the case of overcast sky and partial cloudiness, and in the non-linear approximation (partial cloudiness). In this case it is possible to evaluate the characteristic times of the process and to find the conditions favorable for the appearance of periodic and auto-oscillatory regimes.

II.1. Overcast sky

The equations of heat and water exchange in a plane and horizontally homogeneous system of non-compressible atmosphere over the underlying layer of the ocean, under condition of \( w(o) = O \) (\( W(Z) \) is the velocity of vertical motion) can be written as

\[
-C_p \rho \frac{\partial F_T}{\partial t} - \frac{\partial F_T}{\partial Z} - \frac{\partial F}{\partial Z} - \frac{\partial \tilde{F}}{\partial Z} + L \ m = O \tag{37}
\]

\[
-C_s \rho_s \frac{\partial T_s}{\partial t} - \frac{\partial F_T}{\partial Z} - \frac{\partial F_s}{\partial Z} - \frac{\partial \tilde{F}_s}{\partial Z} = O \tag{38}
\]

\[
-\rho \frac{\partial g_1}{\partial t} - \frac{\partial F_{g_1}}{\partial Z} - m = O \tag{39}
\]

\[
-\rho \frac{\partial g_3}{\partial t} - \frac{\partial F_{g_3}}{\partial Z} + m - m_r = O \tag{40}
\]

Here \( T \) is the temperature of the atmosphere; \( T_s \) is the temperature of the underlying layer; \( g_1 \) and \( g_3 \) are the specific humidity and liquid water amount; \( F \) and \( \tilde{F} \) are the net fluxes of heat and solar radiation in the atmosphere; \( F_s \) and \( \tilde{F}_s \) are the same fluxes in the underlying layer of the ocean. \( (F = F^\uparrow - F^\downarrow; \tilde{F} = \tilde{F}^\uparrow - \tilde{F}^\downarrow \) and the arrows show the direction of radiative fluxes); \( F_T, F_{g_1}, F_{g_3} \) are the turbulent fluxes of heat water vapor and droplet water in the atmosphere; \( F_{T,s} \) is the corresponding heat flux in the underlying layer; \( m \) and \( m_r \) are the velocities of phase transitions of water and precipitation per unit of volume; \( C_p \) and \( C_s \) are the specific heat capacity of the layers; \( \rho \) and \( \rho_s \) their densities; \( L \) is the specific heat.
With $Z = O$ the following conditions are adopted:

$$F^\uparrow - F^\downarrow + \tilde{F}^\uparrow - \tilde{F}^\downarrow + F_T + L F_{g_1} |_{Z = O} = F_S^\uparrow - F_S^\downarrow + F_S^\uparrow - \tilde{F}_S^\downarrow + \tilde{F}_T \mid_{Z = O} = 0$$

(41)

$$F_T \mid_{Z = O} = a (T_s(O) - T(O))$$

(42)

$$F_{g_1} \mid_{Z = O} = b (g_s(O) - g_1(O))$$

(43)

$$F_{g_3} \mid_{Z = O} = O$$

(44)

Here $g_s(O)$ is the specific humidity of saturation under $T_s(O)$, $P(O)$ ($P(Z)$ is the pressure of the air) and $a$ and $b$ are the parameters of the problem. On the upper boundary of the atmosphere with $Z = H$ ($H \approx 8 - 10$ km) we assume

$$F_T(H) = F_{g_1}(H) = F_{g_3}(H) = F^\downarrow(H) = O$$

(45)

With $Z = H_s$, where $H_s$ is the depth of the ocean layer effectively interacting with the atmosphere, we imply

$$F_{T_s}(-H_s) = O$$

(46)

$$F_s^\uparrow(-H_s) = F_s^\downarrow(-H_s)$$

(47)

$$F_s^\uparrow(-H_s) = \tilde{F_s}^\downarrow(-H_s) = O$$

(48)

Here $H_s \approx 50 - 60$ m, the mean depth of attenuation of the monthly temperature wave in the ocean.

The equations (37), (39), (40) are integrated for the height within $(O, H)$ and the equation (38) within $(-H_s, O)$ with application of boundary conditions.
On the basis of experimental studies, carried out by Zubkovsky et al. (1965) the parameters a and b in the formulas (42), (43) can be given in the form

\[ a = a' |\vec{U}_0|; \quad b = \frac{a}{C_p} \]  

(49)

Here \( \vec{U}_0 \) is the velocity of the wind near the earth surface which is one of the outer parameters of the problem.

The constat \( a' \) is chosen so that the formulas (42), (43) would present the mean turbulent fluxes of heat and vapor under mean state of the atmosphere.

The profiles of \( T, T_S, g_1 \) were given in the form

\[ T(Z,t) = T(O,t) - \gamma_o Z \]  

(50)

\[ T_S(Z,t) = T_S(O,t) + T'_S(O,t) \frac{Z}{H_S} \]  

(51)

\[ g_1(Z,t) = g_1(O,t) \exp \left\{ - \frac{2Z}{H} \right\} \]  

(52)

Here \( T'_S(O,t) \) is the derivation of the surface temperature from its equilibrium value and \( \gamma_o = 6.5 \frac{^\circ k}{km} \). Finally the conditions of quasistatics and non-compressibility of the atmosphere are adopted.

The fluxes of solar radiation are written by using the reflecting \( A \) and absorptive \( \pi \) capacities of the atmosphere (the albedo of the ocean is assumed to be equal to zero).

Evidently \( \tilde{F}_{\downarrow}(O) = I_o(1-A)(1-\pi) = I_o(1-A-\pi) \), \( \tilde{F}(H) = I_o(1-A) \), where \( I_o \) is the solar constant averaged over the Earth's globe.

The fluxes of longwave radiation in the approximation (b) (see part I) are presented in the form:

\[ F(H) = F_{\uparrow}(H) = E_S D(m_1, m_3) + E_H (1 - D(m_1 m_3)) \]  

(53)
\[ F(O) = F^\uparrow(O) - F^\downarrow(O) = E_s - E(O)(1 - D(m_1, m_3)) \]  
(54)

\[ E_s = \sigma T_s^4(O); \quad E(O) = \sigma T^4(O); \quad E_H = \sigma(T(O) - \gamma_0 \frac{H}{K})^4 \]  
(55)

where \( \frac{H}{K} \) is the height of homogeneous atmosphere for water vapor 
\((K \approx 3 - 4)\).

For the function \( D(m_1, m_3) \) the analytical expression is used from Feigelson (1970). The albedo of the atmosphere by using the data of Table II can be presented as follows:

\[ A = 1 - K_1 \exp \left\{ -K_2 \tau^k \right\} \]  
(56)

where \( K_1 \approx 0,8 - 0,9, \quad K_2 \approx 0,10 \div 0,12, \quad K_3 \approx 2/3 \) and \( \tau \) is the optical thickness of the clouds (see eq. 22). With \( \tau = 0 \) we obtain \( A = 0.15 \), the mean value of the albedo of the cloudless atmosphere (Avaste et al., 1969).

Finally, the relative absorption of the solar radiation in the atmosphere according to the data of Table II is well approximated by the formula:

\[ \tau = K_4 \log (1,1\sigma'm_3) + [K_5 + K_6(\sigma m_3)^k \gamma] m_1^k m_8 \]  
(57)

Here \( K_4 \approx 6,0 \cdot 10^{-2}, \quad K_5 \approx 8,9 \cdot 10^{-2}, \quad K_6 \approx 7,6 \cdot 10^{-3}, \quad K_7 \approx 0,93, \quad K_8 \approx 0,293. \)

The mean intensity of precipitation \( m_r \) is given in the form (Monin, 1969):

\[ m_r = rm_3 \]

The parameter \( r \) is chosen so that the expression \( m_r = n_0 \rho g_3 H_c \) represents the mean annual quantity of precipitation over the oceans or over the Earth's globe with the mean statistical cloud with thickness \( H_c \), integral liquid water amount \( H_c \rho g_3 \), and cloud amount \( n_0 \).
The condensation rate m is one of the variables of the problem. The system of integrated equations (37)-(40) for the unknown variables is not closed.

To close it the following equation is commonly used: \( g_1(Z,t) = g_{1S}(Z,t) \) where \( g_{1S}(Z,t) \) is the specific humidity of saturation. However, in our formulation the existence of the cloud is not implied. It is possible to use the condition of the radiative balance on the upper boundary of the atmosphere as an additional equation:

\[
\tilde{F}_\downarrow(H) - \tilde{F}_\uparrow(H) - F_\uparrow(H) = \varphi(t) \tag{59}
\]

where \( \varphi(t) \) is a given function of time, determined, for instance, by satellite data. According to Raschke (1968), we can consider \( \varphi(t) \) to be equal to zero when averaged over the globe or over the hemisphere for autumn and spring seasons.

In the linear approximation of the problem we use (59) with \( \varphi(t) = 0 \). In the stationary regime the equation (59) is not independent and we have to set one of the variables as a parameter.

For this purpose we integrate the sum of equations (37)-(39) by height and by time. We obtain then

\[
C_p \int_0^H \rho T dZ + C_s \rho_s \int_0^o T dZ + L \int_0^H \rho g \gamma dZ = \bar{c} = \text{const} \tag{60}
\]

where \( \bar{c} \) is the total heat content of the system.

With the given \( \bar{c} \) the idea of this research is as follows: the equilibrium state and the possibility of periodic processes are considered in the "quasistationary" system, with constant heat content. The second and the third terms in the left part are small if compared with the first one. This has a simple physical sense: ocean is the basic reservoir of heat in the system. Therefore, in the stationary regime we can consider the mean heat content of the active layer of the ocean or the mean temperature of its surface to be the outer parameter of the problem.
The analysis of the equilibrium regime shows that the obtained magnitudes of the equilibrium values have a good correlation with the real mean data.

The deviations from the state of equilibrium are determined by the method of small excitations in the system (37)-(40), (58), (59) (after integration of (37)-(40) by Z).

The solution shows that the character of the process is determined by the reaction of the albedo on variation of liquid water amount \( \frac{\partial A}{\partial m_3} \), by the same reaction of the absorption \( \frac{\partial \pi}{\partial m_3} \) and also by \( \frac{\partial \pi}{\partial m_1} \).

The value of the velocity of the wind \( U_o \) is also significant, i.e., the intensity of turbulent heat and water exchange. The disturbances increase with \( U_o \leq 3 \) m/sec (non-periodic regime) and decrease with \( U_o > 3 \) m/sec (periodic regime with period \( T_{osc} \approx 1 - 2 \) months).

The factors, favorable for sustaining the disturbances, are: increase of air density and height of the tropopause; increase of insolation; decrease of the rate of precipitation, which result in the increase of cloudiness and decrease of \( \frac{\partial A}{\partial m_3} \).

The factors promoting attenuation of the disturbances are: The enhancement of turbulent exchange, increase of the temperature of the ocean and of the values connected with it \( T(O), \frac{\partial g_s(O)}{\partial T_s(O)} \) and the decrease of liquid water amount.

**II.2. Partial cloudiness**

The possibility of long period auto-oscillatory processes is analyzed by Petoukhov (1973), on the same model with an statistically homogeneous cloud system with mean cloud amount and mean integral liquid water amount of clouds \( m_3 \) (so that \( m_3 = n_0 \) \( m_3 \)).

In this case the variables are \( T_s(O,t), T(O,t), g_1(O,t), m_3(t), m(t), m_f(t), n(t) \). To close the system of equations (37)-(40), the empirically defined correlation between the cloud amount \( n \) and mean
relative humidity $f_{rel}$ at a certain atmospheric layer was applied according to Smagorinsky (1960).

$$n = \alpha_o f_{rel} - \beta_o$$  \hspace{1cm} (61)

Here $\alpha_o$ and $\beta_o$ are the parameters depending on the level of clouds. We considered only low clouds ($1000 \text{ mb} \leq p \leq 850 \text{ mb}$).

The following correlation between the integral liquid water amount of clouds and the cloud amount was also used (Smagorinsky, 1960):

$$m_3 = m_{3,0} \ n^{\partial e_o}$$  \hspace{1cm} (62)

where $m_{3,0}$ and $\partial e_o$ are the parameters.

Finally, the seventh equation was obtained on the basis of the analysis of charts of the mean annual quantity of precipitation and the mean annual cloud amount over the oceans (Khrguian, 1953).

$$m_r = m_{r,0} \ \tan (\alpha n^\beta)$$  \hspace{1cm} (63)

where $m_{r,0}$, $\alpha$ and $\beta$ are the parameters taken from the empirical data.

The study of the equilibrium regime shows that with the variation of $U_o$ within $1 - 10 \text{ m/sec}$ the equilibrium values vary within reasonable limits. With the increase of $U_o$, the increase of cloud amount and liquid water amount of clouds takes place. This causes screening of the ocean by cloudiness. The temperature of the ocean falls, and with it the temperature and humidity of the atmosphere.

The equilibrium quantity of cloud amount $n_o$ varies within the range $0.3 - 0.6$ (with $1 \text{ m/sec} \leq U_o \leq 10 \text{ m/sec}$).

The analysis of heat balance in the atmosphere-underlying layer system often involves the discussion of the following problem: which of the two effects of increasing cloudiness dominates? The increase of screening effect of cloudiness, determined by the derivatives $\frac{\partial A}{\partial n}$ and $\frac{\partial A}{\partial m_{3c}}$, or the decrease of the escaping thermal radiation,
described by the derivatives $\frac{\partial F}{\partial n}$ and $\frac{\partial F}{\partial m_3 c}$.

In our case the first two derivatives are summarily greater than the second two, which results in cooling of the system.

The linear non-equilibrium regime with $0.3 \leq n_o \leq 0.6$ turns out to be wave regime. With $0.3 \leq n_o \leq 0.42$ the disturbances have the character of growing wave with period $T_w \approx 2.5 - 3$ months and the characteristic time of increasing of the amplitude by e times $\tau_w \approx 100$ months. With $0.42 < n_o \leq 0.6$, the disturbances are an attenuating wave with $T_w \approx 3$ months and with $\tau_w$ decreasing from $\infty$ with $n_o \approx 0.42$ to $\tau_w \approx 0.6$ months with $n_o \approx 0.6$. With $n_o = 0.42$ the disturbances assume a purely wave character with the period $T_w \approx 3$ months.

It is found that considered equilibrium regime allows auto-oscillatory properties of the system.

We assume the amplitude of oscillations of cloud amount $n'$ to be $0.1-0.3$ and the amplitude of temperature oscillations $T'_S(O,t)$ and $T'(O,t)$ to be about $1^\circ$K. The primary system is reduced to the non-linear equation in relation to $n'$ by rejecting the terms of the second order of smallness as compared with maximum one. Then we receive:

$$\ddot{n'} + 2\delta_o \psi (n', \dot{n'}) \dot{n'} + \omega_o^2 n' = 0$$  \hspace{1cm} (64)

with $\psi (\dot{n'}, n') \cdot |\epsilon| \ll 1$  \hspace{1cm} (65)

Here $n'$ is the derivation of cloud amount $n$ from its equilibrium value $n_o$, $\delta_o$ and $\omega_o$ are parameters depending on the values of equilibrium regime and on parameters of the problem, while $\epsilon = \frac{|\delta_o|}{\omega_o}$. and $\psi (n', \dot{n'})$ is limited. According to the quasi-linear method which
can be adopted in this case. The solution of this equation can be found in the form:

$$n' = a(t) \sin \omega t$$  \hspace{1cm} (66)

It is found that with $0.3 \lesssim n_o \lesssim 0.42$ the disturbances achieve with time the stationary amplitude $a_o \approx 0.1 - 0.2$; in this case $\omega \rightarrow \omega_o$ where $\omega_o$ is the frequency of oscillations in the linear approximation. With $0.42 < n \leq 0.6$ stable equilibrium state with amplitude $a_1 = 0$ takes place (equilibrium regime), and also the unstable limit cycle with the amplitude $a_2 \neq 0$. In this case the disturbances with the initial amplitude $a < a_2$ attenuate with time while the disturbances with the initial amplitude $a > a_2$ increase so that the adopted quasi-linear method of solution of equation becomes inapplicable.

It is interesting to note that in the model the non-linear effects are caused by the reactions of the cloud albedo $A$ and the absorptivity $\pi$ on the liquid water content $m_{3c}$. Usually these reactions are neglected and the equation (64) becomes linear relative to $n'$. In this case there are no stable oscillations with the amplitude of $n' \approx 0.1 - 0.2$ and corresponding amplitudes of $T_s'$ and $T' \approx 1^\circ K$. 
Fig. 1. Cooling out (left) and warming (right) of the stratified cloud: curves 1,3 – the mean experimental data, that is the calculation on the basis of the averaged data of fluxes measurements; 2,4 – the calculation on the basis of the average parameters of the medium.
Fig. 2. Dependence of the albedo of clouds St and Sc on the water content. 1 – experimental points; 2 – curve constructed by the formula (23); 3, 4 and 5 – calculation with $\xi = 0^\circ, 60^\circ$ and $70^\circ$ correspondingly.
Fig. 3: Dependence of $\lambda_{\text{eff}}$ (continuous curves), $\lambda_{\text{m}}$ (dashed lines) on $\tau$ with $\sigma = 50 \text{ km}^{-1}$, $\rho_1 = 5 \text{ g/cm}^3$. Curves: 1 – transmission; 2,3,4 – reflection (curve 2 – $\zeta = 0$; 1, 3 – $\zeta = 60$; 4 – $\zeta = 75^\circ$).
Fig. 4. Dependence of total radiation flux on cloud amount in zenith. Continuous line – experiment, dashed lines – calculations.
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The integral albedo – $A$ and Absorptivity – $\pi$ of cloudy layers in relation to optical thickness – $\tau$ position of the sun – $\mu_0 = \cos \xi$, humidity – $\rho$ with $w = 0.2 \text{ g/m}^3$

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*ρ₁ = 0, the albedo in case of pure scattering.*