THE POWER SPECTRUM OF COSMIC RAY FLUCTUATIONS AND ITS ROLE IN DISCRIMINATING THE STATISTICALLY SIGNIFICANT FLUCTUATIONS IN DETECTION DATA

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RESUMEN

Los resultados de las observaciones de la intensidad de los rayos cósmicos con telescopios de centelleo, supermonitores de neutrones y telescopios de mesones son utilizados para calcular los espectros de potencia de las fluctuaciones de los rayos cósmicos en la superficie terrestre durante varios períodos de actividad solar y geomagnética. Se muestra que la confiabilidad de los cálculos de las estimaciones espectrales relativas, es mucho mayor cuando se hace con base en el espectro calculado.

ABSTRACT

The results of the ground-based observations of the cosmic ray intensity with scintillator telescopes, neutron supermonitors, and muon telescopes are used to calculate the spectra of cosmic ray fluctuation power on the Earth's surface in various periods of solar and geomagnetic activity. It has been shown that the reliability of calculations of the relative sampled spectral estimates is much higher when made on the basis of the calculated spectra.

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The effects of quite a number of processes taking place in the Earth's atmosphere and magnetosphere and in interplanetary space are reflected in the cosmic ray fluctuations inferred from the data of ground-based observations of cosmic ray intensity. It is comparatively easy to take account of the contribution from the atmospheric effects (Dorman, 1975; Dorman, Libin, 1979; Dhanju, 1969) (the procedure of introducing the corrections for pressure and temperature has been properly elaborated). Besides that, the amplitudes of the meteorological fluctuations in cosmic rays are as a rule an order as small as the amplitudes of the fluctuations due to other processes. It is also expedient not only to elucidate the individual role of the other processes responsible for the fluctuation events in cosmic rays (which is rather difficult at the present-day stage of the study of the fluctuations) but also to estimate their total effect.

The results of the ground-based observations of cosmic ray intensity (Dhanju, 1969; Dhanju, 1969a; Attolini, 1975) were used to find the fluctuation spectra which were then compared with the analogous spectra for interplanetary magnetic field. It has been shown (Dorman, 1963) that such spectra in the frequency band \( \sim 10^{-7} \sim 10^{-3} \) Hz may be represented by a function of the form \( f^{-\gamma} \) where \( 1.5 \leq \gamma \leq 2.0 \) for cosmic rays. It has also been shown (Attolini, 1975) that the function may be extended up to frequencies \( f \geq 10^{-2} \) Hz. (The power spectra may also be presented as \( [1 + (2\pi f T)^2]^{-1} \), where \( T \leq 5-6 \) days. Such representation, however, is not quite justified since the fluctuation spectrum for interplanetary magnetic field can be adequately described by the dependence \( f^{-\gamma} \) (McCoy, 1967). Fig. 1 presents the power spectra of cosmic ray fluctuations in the frequency band \( 6 \times 10^{-6} \leq f \leq 10^{-3} \) Hz calculated on the basis of the hourly and 5-minute values of cosmic ray intensity for 1978-1979 obtained with a neutron supermonitor, muon telescope, and a scintillator telescope at Moscow and Bologna. The figure also shows the results of the calculations of the sampled spectral estimates (Attolini, 1975) in the frequency band \( 6 \times 10^{-7} \leq f \leq 2 \times 10^{-4} \text{Hz} \) carried out on the basis of the observation data obtained with neutron supermonitors at Leeds, Kiel, Deep River, Alert, Calgary, and Sulphur Mountain and the scintillation telescopes at Chacaltaya and Bologna for 1966-1968. It can be seen from the figure that, within the errors (the correlation coefficient is about 0.94), the fluctuation spectrum may be presented in the form \( f^{-\gamma} \), where \( \gamma = 1.956 \pm 0.089 \). Also within the errors, the spectra of the fluctuations in the scintillation telescope data at Moscow and Bologna are the same as the value obtained above; for them, the power-law exponent of the spectrum is 1.918 and 1.977, respectively. (It should be noted that the results of all the calculations of the sampled spectral estimates were reduced to the mean value \( C_{xx} \) at \( f \sim 8.5 \times 10^{-5} \) Hz). A somewhat different pattern is observed for the spectra calculated for relatively short periods. Table 1 lists the values of the spectral power-law exponent \( \gamma \) of cosmic ray fluctuations in the spectra calculated on the basis of the data of observations of the general ioniz-
ing component obtained with the IZMIRAN scintillator supertelescope for 4-day and 20-day periods in September-October, 1978. The presented results are indicative of a certain knee of the spectrum in the frequency band $f \geq 2 \times 10^{-4}$ Hz, which is qualitatively confirmed by the calculations of the spectra for November 24-December 20, 1965, February 22-March 20, 1966, and May 20-June 15, 1966 (Dorman, 1975; Dorman, Libin, 1979; Dhanju, 1969a). The nature of the observed knee needs being studied further (in the work of Attolini (1975), the spectral power-law exponent $\gamma$ is constant up to $f \leq 10^{-3}$ Hz). Nevertheless, the conclusion may be draw even now that the contribution from white noise to the power of the sampled spectral estimates in the high-frequency-band may be one of the factors responsible for the observed knee. In fact, the rigidity of the particles scattered resonantly by inhomogeneities of scale $\ell$ may be determined from the relation

$$ R \approx 300 \, H \times \ell $$

where $H$ is the magnetic field intensity; $\ell = u/f$ is the characteristic size of inhomogeneities; and $u$ the solar wind velocity. Table 2 presents the mean values of the inhomogeneity sizes $\ell$ and the rigidities $R$ of the particles scattered resonantly by the inhomogeneities (whose sizes are proportional to the Larmor radius of particles) for the cosmic ray fluctuations of various periods, at solar wind velocity $u = 4 \times 10^7$ cm/s and interplanetary field intensity $H = 5 \times 10^{-5}$ - $10^{-4}$ gauss. The results obtained show that the particles with rigidities $R \leq 3$ GV play the major part in the cosmic ray fluctuations at $f > 2 \times 10^{-4}$ Hz and that the contribution from such particles to the cosmic ray intensity detected on the Earth does not exceed $1 - 1.5$ of the total number of particles. (As a first approximation, the power of the sampled spectral estimates of cosmic-ray fluctuations in the frequency band $f \gtrsim 10^{-4}$ Hz may be considered proportional to the coupling coefficients of the scintillator supertelescope presented in Fig. 2). At the same time, the fluctuations associated with the statistical nature of the detected radiation and, partly, with instrumental noise (the “white” noise) are practically frequency-independent, so the total effect of superposition of the true “fluctuations and those due to the noise give rise to the knee (flattening) of the spectrum. The flattening of the spectrum at $f > 2 \times 10^{-4}$ Hz may also be accounted for by the fluctuations of the geomagnetic cutoff threshold to which the particles with rigidities $R \leq 3$ GV are particularly sensitive. If the contribution from “white” noise is only taken into account, the values of $\gamma$ (from Table 1) in the high-frequency band will be about $1.7 - 1.8$, which fails to completely restore the observed spectrum. Nevertheless, the result obtained confirms the conclusion (Dorman, 1979) that the spectrum of cosmic ray fluctuation power is constant up to frequencies $f \lesssim 10^{-2}$ Hz.

It is of special interest, in studying the modulation effects, to examine the
frequency dependence of the spectral power-law exponent of the cosmic ray fluctuation power for various periods of solar activity (Owens, 1973). The preliminary results of the analysis of the spectra of the general ionizing, neutron and muon components of cosmic rays calculated for September-October, 1969, September-October, 1974 and February-June and September-October, 1978 (Fig.1) have shown that the spectral power-law exponent $\gamma$ was about 1.9 - 2.1 (including the calculation errors) near the solar maxima of 1968-1969 and 1978 and 1.7 - 1.8 during solar minima.

The result obtained is in good agreement with the spectra calculated elsewhere (Dhanju, 1969; Dhanju, 1969a; Attolini, 1975; Owens, 1973). Though the calculation results obtained need being further detailed on the basis of a layer volume of the analyzed data, they may nevertheless be used also to calculate the relative sampled spectral estimates of $\tilde{A}_{XX}$:

$$\tilde{A}_{XX} = \frac{\tilde{C}_{XX} - \Gamma_{XX}}{\Gamma_{XX}}$$

where $\Gamma_{XX}$ is the cosmic ray fluctuation spectrum set by the parametric function of the form $f^{-a\gamma_X}$. Here, the parameter $a$ is also frequency dependent, namely

$$a = \frac{\log f}{\log f + 2}$$

in the frequency band $10^{-4} \leq f \leq 1.66 \times 10^{-3}$ Hz.

Fig. 3 shows the spectral power law exponent $\gamma_X$ as a function of frequency for the period from September 20 to October 2, 1978. It can readily be seen that the presented dependences of $\gamma_X$ do not contradict the results obtained earlier.

In fact, the values of $a\gamma_X$ at, for example $f \sim 10^{-4}$ Hz are about 1.7 - 2.0 and are quite in agreement with the values of $\gamma$ listed in Table 1. The use of the dependences of the type $f^{-a\gamma}$ in calculating the sampled relative spectral estimates makes it possible to extend significantly the potentialities of the analysis of the obtained spectra. Fig. 4 presents the relative spectra of cosmic ray fluctuations for each day in September-October, 1978. It can be seen that the selected model permits the information about the main peaks exceeding the reliability interval to be derived from the calculated spectra (Dorman, 1979) (in the figure, the 95% level of confidence is about 3.5 relative units). The isolated peaks with periods of about 50,
20, and 17 min are observable for September 23, 25, and 29 (the analysis was made in high-frequency band). Application of the selected model permits reliable results to be also obtained in low-frequency band. Fig. 4 presents the relative sampled spectral estimates which show that the fine structure of the calculated spectra has been properly resolved.

### Table I

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>20</th>
<th>23.09</th>
<th>24 – 27.09</th>
<th>28.09 – 1.10</th>
<th>2.10 – 5.10</th>
<th>6.10</th>
<th>20.09 – 9.10</th>
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<tbody>
<tr>
<td>$10^{-5} \leq f \leq$</td>
<td>1.74±</td>
<td>1.81±</td>
<td>1.98±</td>
<td>1.65±</td>
<td>1.81±</td>
<td>1.977±</td>
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<tr>
<td>$10^{-4} \leq f \leq$</td>
<td>0.19</td>
<td>0.17</td>
<td>0.13</td>
<td>0.19</td>
<td>0.19</td>
<td>0.098</td>
<td></td>
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<tr>
<td>$10^{-4} \leq f \leq$</td>
<td>1.06±</td>
<td>0.97±</td>
<td>0.98±</td>
<td>0.85±</td>
<td>0.77±</td>
<td>1.271±</td>
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<tr>
<td>$10^{-3} \leq f \leq$</td>
<td>0.21</td>
<td>0.14</td>
<td>0.10</td>
<td>0.18</td>
<td>0.05</td>
<td>0.074</td>
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### Table II

<table>
<thead>
<tr>
<th>$f$, Hz</th>
<th>$9.2 \times 10^{-5}$</th>
<th>$1.4 \times 10^{-4}$</th>
<th>$2.8 \times 10^{-4}$</th>
<th>$4.2 \times 10^{-4}$</th>
<th>$8.3 \times 10^{-4}$</th>
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<tr>
<td>$\ell_c$, cm</td>
<td>$4.3 \times 10^{11}$</td>
<td>$2.86 \times 10^{11}$</td>
<td>$1.43 \times 10^{11}$</td>
<td>$9.5 \times 10^{10}$</td>
<td>$4.8 \times 10^{10}$</td>
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<tr>
<td>$R_e$, V</td>
<td>$12.9 \times 10^9$</td>
<td>$8.6 \times 10^9$</td>
<td>$4.3 \times 10^9$</td>
<td>$2.9 \times 10^9$</td>
<td>$1.5 \times 10^9$</td>
</tr>
</tbody>
</table>
Fig. 1  The ground based spectrum of cosmic ray fluctuation power in the periods near solar maxima.
Fig. 2 The coupling coefficients of scintillator supertelescope.

Fig. 3 The exponent of cosmic ray fluctuations spectrum versus frequency from September 20 to October 2, 1978.
Fig. 4  The relative spectra of cosmic ray fluctuations for each day of the period from September 20 to October 2, 1978.
ATTOLINI, M. R., S. CECCINI, I. GUIDI, M. YALLI, 1975. The shape of the power spectrum of cosmic ray at ground level up to 7.10^{-3} Hz. Planet and Space Sci., 23, No. 12, 1603.


